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GeT: The News!

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Instructional Capacity for High School Geometry

by Pat Herbst

Welcome to our second issue of *GeT: The News!* This newsletter, published three times a year, helps us connect to and communicate with members of GeT: A Pencil, an inter-institutional support network for instructors of Geometry for Teachers (GeT) courses and high school geometry teachers. The network was developed with the goal to provide support for collective stewardship and connection-building across individual instructors. It is housed at the GRIP Lab at the University of Michigan School of Education.

In the new year, we have continued to sponsor two monthly seminars for members of our community, starting with presentations by Claudine Margolis and Mollee Shultz that reported on interviews they have been conducting with individuals whose work connects in one way or another to geometry for teachers courses. On January 24, Claudine shared preliminary interview data from secondary school leaders in Michigan, including high school mathematics department chairs, state and district mathematics leaders, and school principals. On February 4, Mollee shared information about how department administrators at the college level staff geometry courses. (Click on a date to watch a recording of either seminar.)

These interviews support the more general aim of identifying the sources of the problem that we have been describing as the need to improve instructional capacity for teaching high school geometry. As instructors of geometry for future secondary teachers, members of GeT: A Pencil are part of a larger system responsible for supporting students' learning of mathematics at the secondary level. High school geometry plays an important role in students' mathematics education: not only does it provide them with a distinct "language" of representation that they can use to visualize real world problems and abstract ideas, but it can also bring students closer to theoretical mathematics as it involves them in practices such as conjecturing and proving. As such, high school geometry can play an important role in recruiting students for STEM careers. If we accept that the quality of students' mathematical experiences depends at least in part on the quality and quantity of their teachers' knowledge, it would follow that the quality of future teachers' experiences in geometry courses for teachers might have consequences for high school students. As those high school students continue through their studies, good experiences in mathematics courses might support their decisions to study more

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Contributed Essay

Teaching for Understanding

by Sharon Vestal

Geometry for Teachers (GeT) is a course at South Dakota State University (SDSU) that is entirely made up of mathematics majors who are planning to become certified to teach middle and high school mathematics. It is typically the first mathematics course that our preservice teachers take that includes pedagogy. Since I want the students to be well prepared to teach high school geometry, we focus on Euclidean geometry throughout the course.

This fall when I walked into class on the first day, I heard one student say to another, "we just need to memorize it." I informed the student that the word "memorize" was not to be used in my classroom—our goal should always be to understand mathematical concepts and to teach our students in a way that develops their understanding. In this article, I will outline some discovery activities that I used with my GeT students to help them understand formulas rather than relying on memorization of formulas. My goal is that they will use these experiences when they are teachers to help their own students learn with understanding rather than just memorizing.

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Instructional Capacity - continued from pg. 1

mathematics in college, or perhaps even to consider mathematics teaching themselves. But of course, things may or may not turn out that way: unprepared geometry teachers may also teach courses devoid of any interest, turning students off to the pursuit of more mathematics. Instructional capacity seems like it should be a key factor to consider when staffing high school geometry courses as well as other high school mathematics courses. We have been wondering who the players in the K-12 and college levels are that can contribute to understanding the problem of instructional capacity.

The first players we identified were members of GeT: A Pencil, and we have been interviewing each of you to understand how you think about the undergraduate geometry course for teachers. Gleanings from those interviews have suggested to us other stakeholders to reach out to. Clearly, mathematics faculty who teach GeT courses are not responsible alone for teacher preparation; education faculty and staff also play a role and we are beginning to interview them in search of insights on how they think of their role in improving instructional capacity for teaching geometry. In particular, we are speaking to mathematics department administrators who have to find instructors for GeT courses, asking what they look for in an instructor and how easy it is for them to find one. Cognizant that staffing may also have some difficulties at the high school levels, our interviews of high school mathematics department chairs and district mathematics coordinators have included questions such as how easy it is for them to find teachers who can and want to teach high school geometry. Finally, we have reached out to college graduates who took GeT courses as undergraduates and later joined the teaching workforce. We have asked them whether and how they've felt prepared and excited to teach high school geometry, and how those sentiments connect with their experiences in GeT courses.

These interviews are helping us conceive of surveys that we'd like to distribute more widely to gauge the extent to which instructional capacity for teaching geometry is a problem. We would welcome your input in this project: Do you have ideas for topics or questions to include in these surveys? Or do you have suggestions of other groups of stakeholders who might have perspectives that are useful to collect? Submit any thoughts or ideas about the survey questionnaire here.

Enjoy this newsletter and consider contributing an essay yourself (details are on pg. 5). I know we'll see some of you at the RUME conference in Boston on February 27. Looking forward to it! I hope that the Spring semester continues to bring you professional satisfaction, and that you are able to take some time off to enjoy yourself during Spring Break!

GeT Activity

Exploration, Construction, and Proof as Resources for Teaching Geometry Through Problems

by Claudine Margolis

As the call to teach mathematics through collaborative problem-solving reaches a wider audience, more secondary school teachers are grappling with the difficulties inherent in facilitating learning through problems. Open problems provide opportunities for students to engage in authentic disciplinary practices such as formulating and evaluating conjectures, considering the costs and affordances of various problem-solving approaches, and using mathematical justifications to support claims. These benefits to student learning come at a cost to the instructor, who has to grapple with facilitation decisions that have mathematical and pedagogical implications. One common difficulty with facilitating student learning through open problems is that students can have trouble getting started on the problem, especially if they are used to more traditional problem formulations that provide clearer hints as to the expected solution method.

One strategy for supporting students' work on open problems is to provide instructional cues that indicate the kind of mathematical work expected on a problem. The Pool Problem Activity cues students into three kinds of instructional situations—Exploration, Construction, and Proof—to support student thinking and discourse while maintaining the benefits of an open problem. The activity was built with Desmos' Activity Builder which has several built-in features that support mathematical discourse and facilitation of whole-class discussions.

The Pool Problem

Three swimmers have arranged a race to win $314. A buoy will be placed somewhere within a rectangular pool, and the swimmer that reaches it first wins the prize money. One swimmer will start from a corner of the pool, and the other two will start from somewhere along the adjacent sides. They need your help to determine where the buoy should be placed so that the competition is fair.

Follow along online! Go to this link and click "Student Preview" to work through the screens.

The activity begins with a dynamic exploration of the pool problem. Students can choose starting positions for the swimmers, experiment with buoy placement, and then watch as the swimmers race toward the buoy at a steady pace. Students build intuition through the exploration before advancing to the next screen where they write about the geometric relationship between the position of the swimmers and the buoy.

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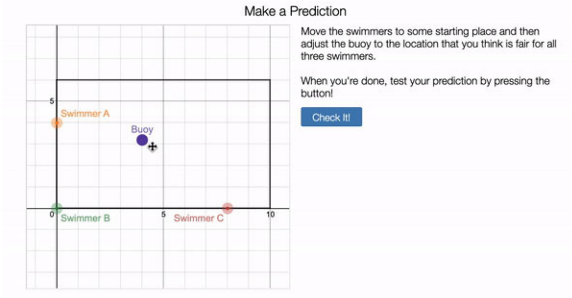
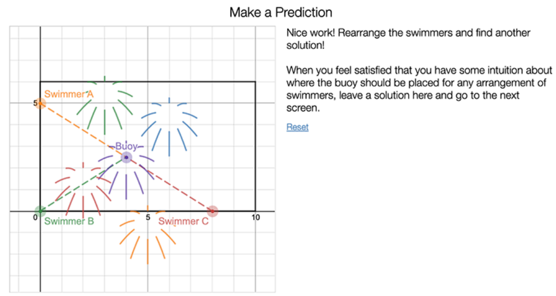
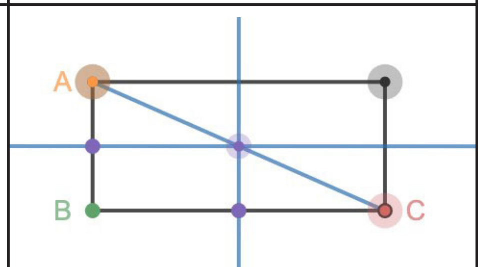
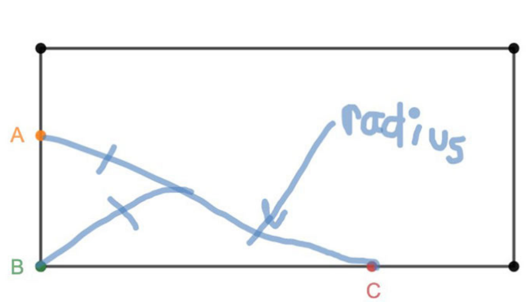
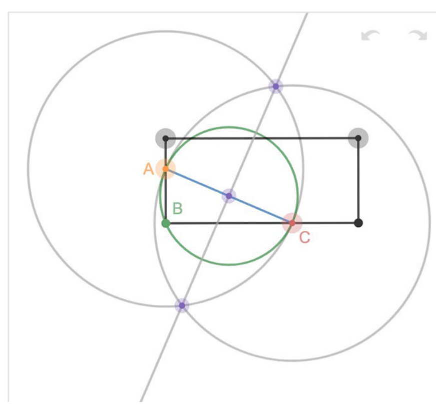
Teaching through Problems - continued from pg. 2





After they have developed some intuition and informal understanding of the geometric relationships, students advance to the construction portion of the activity. First, they plan a construction strategy by sketching or describing the geometric relationships they intend to leverage in order to construct the exact location of the buoy. Next, they are asked to use dynamic geometry construction tools to carry out their plan. The plans and constructions below were made by two pairs of participants who worked together on this activity at a teacher conference.

Plans made by each pair	Constructions made by each pair
Construct line segment AC. Construct midpoint M, by constructing circles A and C with radius AC or CA. Then construct circle M with radius AM. Does this circle pass through B? If so, equidistant!	
	

In the final stage of the activity, students are asked to brainstorm as many relevant conjectures as they can think of (see examples below), and then asked to select one of the conjectures to formally prove.

The Pool Problem Activity provides opportunities for students to engage in a range of authentic mathematical practices within a single context. If this activity is used within a GeT course, pre-service teachers will experience the benefits of each instructional situation (Exploration, Construction, and Proof) for their own learning and may be more likely to provide their future students with opportunities to engage with rich, open mathematical tasks.

What is Desmos?

Desmos' Classroom Activity platform gives the instructor access to each student's progress throughout the activity. In addition to built-in features that allow students to see a sample of responses from their classmates, there are several Classroom Conversation features that support whole-class discussion at any point in the activity. Instructors have the ability to restrict students to certain screens (with the pacing feature), pause the activity for all students, and anonymize student names for whole-class projection.

Learn more about Desmos Classroom Activities.

Conjecture(s)
The hypotenuse of a right triangle contains the center of the three vertices.
The perpendicular bisectors of a triangle meet on the hypotenuse.
The midpoint of the hypotenuse can create two isosceles triangles.

Conjecture(s)
The hypotenuse of a right triangle is where the buoy should be located
The perpendicular bisectors of the legs of the right triangle intersect at the buoy.
The midpoint of the hypotenuse is equidistant to the three vertices of the right triangle.

Claudine Margolis is a research assistant in the GRIP Lab.

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Sum of Angles in a Triangle

Throughout the semester my students used the theorem for the sum of angles in a triangle, but they didn't really understand how we knew it. So in class, I gave each of my students a half-sheet of paper and asked them to use a straightedge to draw a triangle and a quadrilateral and to cut out their figures. Next, I asked them to cut or tear the corners as shown in Figures 1 and 2. Once they had torn off the angles, I asked them to put the vertices of the triangle together so that they are touching. With the triangle, the students immediately saw that together these angles formed a straight angle, which demonstrates that the sum of the measures of the angles in a triangle is 180°. I had them repeat this same process with the quadrilateral in order to recognize that the sum of the measures of the angles in a quadrilateral is 360°. While this was a quick activity to do with the students and required few materials (half sheet of paper, straightedge, and scissors), it gave them a physical representation of facts that they had been using throughout the semester.

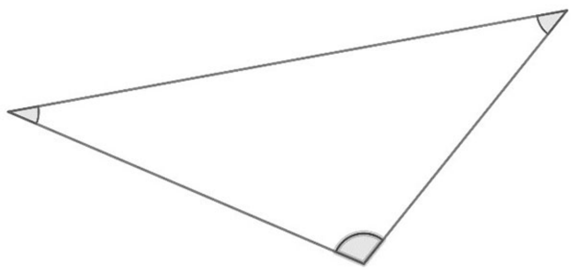

Figure 1

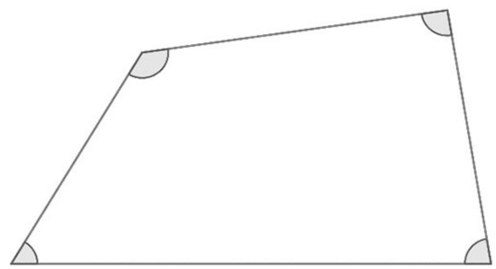

Figure 2

Area Formulas

Another discovery activity that I have used involves finding the areas of rectangles, triangles, parallelograms, and trapezoids using only the fact that the area of a rectangle is base*height. I gave students a piece of cardstock with Figures 3, 4, 5, and 6 printed on it, and asked them to cut out each of the figures. Then I had them take the rectangle (shown in Figure 3) and cut along the diagonal so that they could "see" that the area of a triangle is ½ base*height.

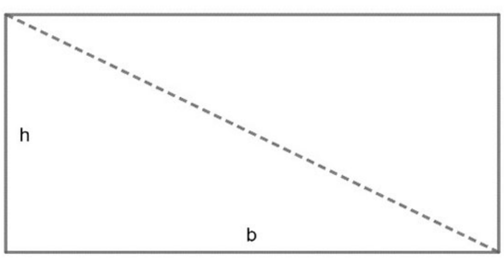

Figure 3

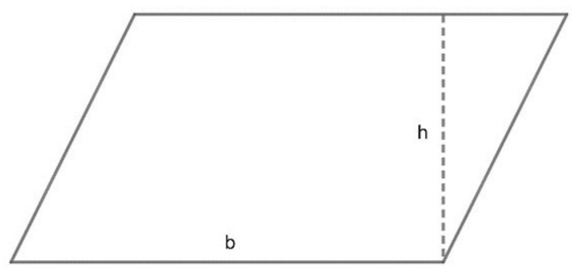

Figure 4

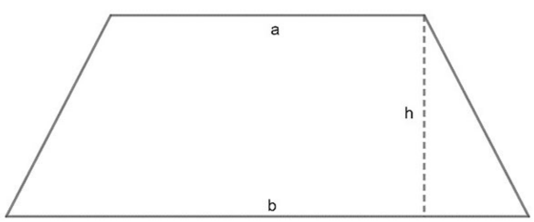

Figure 5

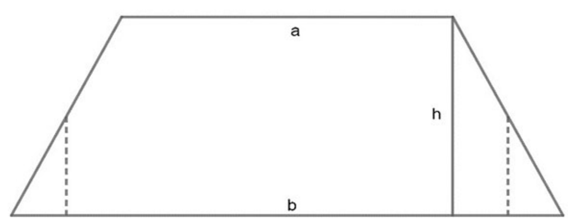

Figure 6

Next, they cut the parallelogram (shown in Figure 4) along the dotted line. Then they translated this right triangle to the left so that the hypotenuse lined up with the side of the parallelogram, creating a rectangle. Looking at this rectangle, they saw why the area formula for a parallelogram is base*height.

To understand the formula for the area of a trapezoid, we looked at it in two ways. Using the trapezoid shown in Figure 5, students cut along the dotted line and reflected the triangle over the bottom base of the trapezoid. Next, they translated the triangle left and vertically so the hypotenuse of the triangle lined up with the non-parallel side of the trapezoid, forming another rectangle. The rectangle clearly had height h, but the length of the base of the rectangle wasn't obvious. They observed that the length of the base was a number between a and b, and then eventually came up with the base length of $\frac{(a+b)}{2}$, the average of the bases. Again using the formula for the area of a rectangle, they

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concluded that the formula for the area of a trapezoid is $A=\frac{(a+b)}{2}\cdot h$

Now using Figure 6 and cutting along the dotted lines, they created another rectangle by rotating these small right triangles 180° about the midpoints of the non-parallel sides of the trapezoid. Once again, they created a rectangle and "saw" the formula for the area of a trapezoid.

Distance Formula

When we started the discussion of the distance formula, I asked my students how the formula was explained to them. Some of my GeT students said that their teacher wrote the formula on the board and told them to memorize it: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$. Again, this idea of having students memorize formulas without understanding them is not what we want our future teachers doing. So, I plotted the points (x_1,y_1) and (x_2,y_2) on a coordinate plane, drew the right triangle (shown in Figure 7), and marked the hypotenuse of the right triangle, *d*.

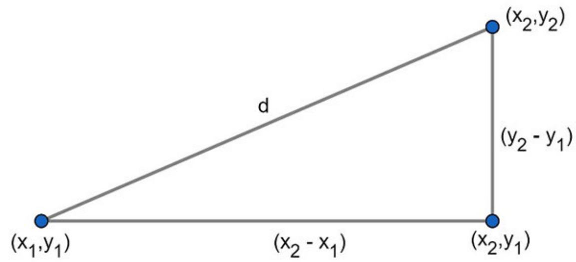


Figure 7

Then the students found the lengths of the legs of the right triangle and used the Pythagorean Theorem, giving them:

$$(x_2-x_1)^2+(y_2-y_1)^2=d^2$$
$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=d.$$

Now rather than memorizing the distance formula, my GeT students understood its origin.

Using these discovery activities in my GeT course to cover basic concepts in geometry gave my students active learning strategies to use in their own classroom. For some of these students, it was the first time that they experienced active learning in a mathematics course. In addition, completing these exercises illustrated the importance of teaching mathematics for understanding rather than telling students to memorize formulas. Mathematics education research indicates that memorizers are the lowest achievers in mathematics (Boaler, 2015).

Throughout our mathematics courses for preservice teachers, we need to model best practices. These discovery activities facilitate meaningful mathematical discourse, connect mathematical representations, and build procedural fluency from conceptual understanding, which are some of the mathematics teaching practices found in NCTM's *Principles to Actions* (2014). As we prepare future mathematics teachers, we need them to understand the importance of what they do every day and the impact that they have on their students' learning. Many of my GeT students dislike geometry at the beginning of my course, frequently because they had had a bad experience in their high school geometry course. By the end of the course, most feel prepared to teach geometry and some even enjoy geometry.

Sharon Vestal is an Associate Professor at South Dakota State University

Citations

Boaler, J. (2015, May 7). Memorizers are the lowest achievers and other Common Core surprises. *The Hechinger Report*.

National Council of Teachers of Mathematics. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: Author.

Contribute an essay

We welcome contributions from members of the GeT: A Pencil community! Activities you tried in class, things you observed your students do, reflections on your experience teaching, thoughts on what the GeT course should include. . . any of these and others would be fair game to write about. Consider the length of the articles in this issue as examples of how long your piece could be. To pitch your idea, email us at **GRIP@umich.edu**.

GeT: A Pencil Working Group Updates

Teaching GeT (led by Nat Miller)
Unlike more standardized classes such as calculus, future instructors of the collegiate geometry course for pre-service teachers are assigned to the course with vastly different geometry experiences, and many of them have never taken a post-secondary geometry course at all. The Teaching GeT (Geometry for Teachers) working group was formed to conceive and draft written products that could fill in such gaps, focused on answering the core question: What would be useful for a new GeT instructor to consider prior to teaching the GeT course?

During the Fall 2019 semester, we put together a long list of potential topics that we could address. So far, we have mainly focused on one in particular: Trying to come up with a list of Essential Student Learning Outcomes (SLOs) that most people would agree should be addressed in any GeT course. This is closely related to the work that the Geometry Knowledge for Teaching workgroup did last year, looking at SLOs for some of the different types of GeT courses identified by Grover and Connor (2000). Our hope is to get feedback about which SLOs are seen as truly essential from the perspective of the wider community of people teaching GeT courses, so you may get a short survey from us sometime in the near future!

Going forward, we hope to identify the skills and knowledge necessary to teach our Essential SLOs that potential teachers may have not yet obtained and to think about ways to help them acquire these. One approach we may take is trying to write guides to some particular content or pedagogical areas that we think are of particular concern. We would welcome participation from anyone interested in helping with this project, even those who haven't been previously involved!

What's happening?

February 27 (Thursday), 8am-12pm ET
GeT: A Pencil Working Group at the RUME 2020 Conference (Boston, MA)

March 13 (Friday), 4pm ET
GeT Seminar (online): *Geometry and the Visual Arts: What can we learn from teachers' perspectives about textbook problems?* (Gloriana González & Christine Rinkerberger)

April 17 (Friday), 12-1:30pm PT
The GRIP Lab team presents research on MKT-G growth at the AERA 2020 Conference (San Francisco, CA)

To list an event in an upcoming newsletter, email us at **GRIP@umich.edu**.

Transformations (led by Julia St. Goar)
In the fall we articulated our main goals for the Transformations group: (1) to create a system of axioms for transformation proofs; (2) to get ideas for improving existing courses for future teachers involving transformations; (3) to grapple with what teachers really need to know about transformations in the Common Core and how we can motivate their study of transformations; (4) to make activities for the purposes of teaching transformation to undergraduate geometry students; (5) to understand axioms in a transformation context so that teachers can compare these axioms to other contexts; (6) to get help creating transformation content for a course for the first time; and (7) to deepen our understanding of axiomatic structures in a transformation context.

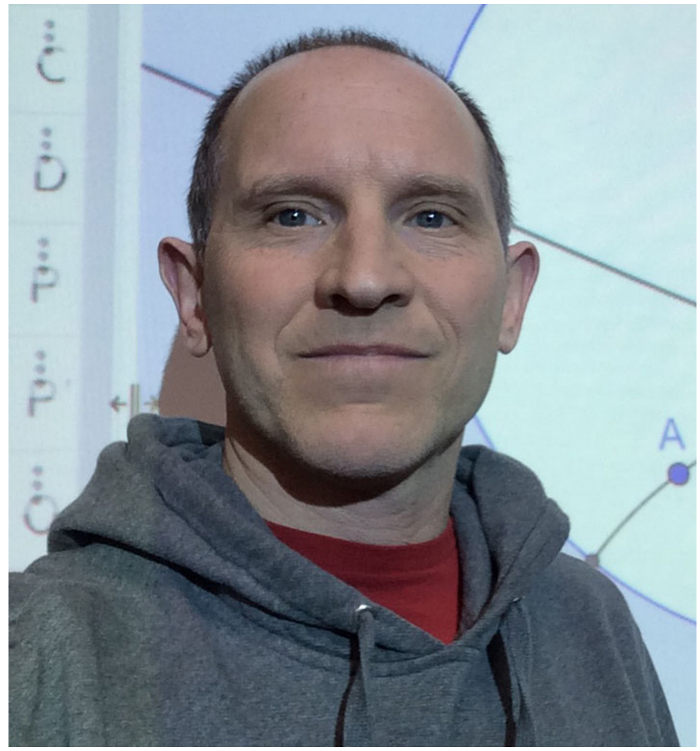
So far, we have shared a list of content goals each of us has in our own courses, including key concepts and definitions as well as theorems proved. This resulted in the beginning of a content concept map for transformation courses. We discussed classroom activities, teaching strategies, and technological tools each of us has been using to further student understanding of transformations, especially for the topics that our students have found tricky. We also shared information on other aspects of our courses, such as student audience composition, prerequisites, textbooks used, teaching goals, and even classroom culture, in order to discover the similarities and differences.

We have debated our goals for any axiomatic system we may create. One question was whether the axiomatic system should be adjusted for the undergraduate student audience (for example, a statement that is really a theorem may be called an axiom in an undergraduate classroom context). Axioms could be written to create a smoother classroom experience, to avoid having to put students through proving too many very basic theorems, and to allow students to get to more interesting theorems more quickly. Properties that high school students have learned about transformations in the past, for example, could potentially be considered axioms in a classroom setting. It became the view of the group that creating a student-friendly axiomatic system would be the most helpful for our purposes.

This spring we have begun by sharing various existing axiomatic systems to compare and contrast. It is likely that our discussion will focus on: the appropriateness of the axiom statements themselves in a classroom context, particularly for student understanding; the connection of these axioms to the Common Core; and the theorems that these axioms will allow students and teachers to prove throughout the course. This discussion on axioms will likely reinforce our ongoing discussion of ways to help undergraduate student understanding and to develop classroom activities focused on transformations. The group would welcome any new voices that could help us in these ongoing efforts.

GeT to know the community

Four questions with **Stephen Szydlik**, Professor of Mathematics at the University of Wisconsin Oshkosh



- **What is special about your GeT course?** My GeT course is axioms-based, but I blend the rigorous mathematics with opportunities for exploration and active learning. I try to give my students opportunities to model authentic mathematical behaviors: investigation, conjecture, counterexample and logical argument. I emphasize proof, but we spend lots of time working with the hyperbolic models, especially using dynamic geometry software.

- **Who are your students?** The students in my GeT course are almost entirely preservice secondary teachers. Most come from within Wisconsin, and many have never been outside the state. I try to encourage them to travel whenever possible, and just as stepping outside their culture provides them with a unique perspective on their homes, so too does our investigation of hyperbolic geometry offer new insights into the Euclidean geometry that they will teach. (At least that's my goal!)

- **What are you most interested in learning/achieving through participating with the GeT: A Pencil community?** I've already learned so much from the GeT: A Pencil community about the many different ways of structuring a GeT course! I am most interested in learning how well we prepare our future teachers and if there are ways that we can better serve them.

- **What is your favorite book you have read recently?** I just finished Fredrik Backman's novel *Us Against You*. At its most basic level, it's a story of an economically depressed northern town struggling to heal after an act of violence. The characters are complicated and richly drawn, and the story, centered around the reemergence of the town's hockey team, is riveting and emotional. It was a rewarding read!

Did you get promoted? Win a grant? Have a baby? Buy a house? We would love to feature your news, whether professional or personal! Email us at **GRIP@umich.edu**.

Recent Publications

Members of GeT: A Pencil have recently published in a diverse set of journals. Priya Prasad's team wrote for *The Mathematics Enthusiast*, Orly Buchbinder co-authored a paper that appeared in JRME, and Tuyin An and her colleague presented at the Interdisciplinary STEM Teaching and Learning Conference. In the more mathematical realm, Michael Ruddy is awaiting the publication of his paper on linear Gaussian covariance models. Congratulations to everyone! **To submit a paper to be highlighted in a future newsletter, please fill out this form.**

- **An, T.**, & Nguyen, H. (2018) Incorporating the dragging feature of dynamic geometry environments in teaching and learning college geometry. *Proceedings of the Interdisciplinary STEM Teaching and Learning Conference*, Vol. 2. doi: 10.20429/stem.2018.020107

- **Buchbinder, O.**, Chazan, D. I., & Capozzoli, M. (2019). Solving equations: Exploring instructional exchanges as lenses to understand teaching and its resistance to reform. *Journal for Research in Mathematics Education, 50*(1), 51–83. doi: 10.5951/jresematheduc.50.1.0051

- Castro Superfine, A., **Prasad, P. V.**, Welder, R. M., Olanoff, D., & Eubanks-Turner, C. (2020). Exploring mathematical knowledge for teaching teachers: Supporting prospective teachers' relearning of mathematics. *The Mathematics Enthusiast*, *17*(2–3), 367–402.

- Coons, J. I., Marigliano, O., & **Ruddy, M.** (2019). Maximum likelihood degree of the small linear Gaussian covariance model. arXiv preprint arXiv: 1909.04553

π Day by Amanda Milewski

When I first began my career as a high school geometry teacher (in 2000), neither I nor my colleagues had ever heard of "π Day". In perusing one of the many practitioner journals, I learned about other secondary mathematics teachers celebrating March 14th with their students in a variety of ways, including pie-eating and baking contests as well as school-wide recitations of the digits of π. As a new teacher, I decided my students and I would join in the celebration. In that first year, only a handful of students seemed excited to participate, but by the time the second year rolled around, many more of my students had heard about the event and were on board. This enthusiasm for the celebration had its way of catching the attention of my peers, and by the time I reached my third year of teaching, I had two additional colleagues join in on the fun with their students.

It has been over a decade since I have worked as a high school teacher and I wonder sometimes what "π Day" looks like now, after it has had some time to gain prominence. It is certain that the public's awareness of π Day has grown. For example, for some time now, my friends and family members (otherwise unaffiliated with mathematics) have taken it upon themselves to wish me a "Happy π Day". On more than one occasion, one of these well-wishers has sent a pie to my home (which quickly disappeared as soon as my boys doubled down on the celebration). A few years ago, I and others at the GRIP lab participated in a π Day 5K (or 3.1 mile) run, though I opted out of the pie-eating contest that strangely came *before* the run.

Beyond the general public's awareness, I have also noticed local and national pie and pizza companies cashing in with their homages to π. With so much hype about the food-related activities of π Day, I have found myself wondering a bit about whether mathematics has taken a back seat to. . . well, eating pie!? While I am not opposed to adding another food holiday to the calendar, I do find myself hoping that the food celebration does not ultimately distract from the opportunity for the general public to grow its appreciation of mathematics.

Perhaps I am not the only one with this concern. This year, the United Nations Educational, Scientific and Cultural Organization (UNESCO) has declared March 14 as International Day of Mathematics. Notably, π is missing from this title. With this declaration, I wonder whether UNESCO is asking the pie to take a back seat to the mathematics? And if so, I am all for it. But does this mean I won't be getting pie sent to me this year? On second thought, maybe we should keep it π day after all!

Do you celebrate π Day with your students? If so, please send us a note and/or picture that we can share with the community in the next newsletter!

Members of the GRIP Lab complete a π Day 5k

GeT Support

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Patricio Herbst, PI
Amanda Milewski, Co-PI
Erin Lichtenstein, *GeT: The News* Editor

GeT Support is housed in the GRIP Lab at the University of Michigan
Inese Berzina Pitcher, Project Manager

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