# GeT: The News!

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## Developing and Stewarding the Get: A Pencil Community

by Amanda Brown

One of the most commonly discussed goals of the GeT Support Project is working collectively to improve the instructional capacity for high school geometry. It makes sense for this to be a point of common discussion for us, as it is the goal to which the Get: A Pencil community is collectively committed. There is, however, a second, equally important goal of the GeT Support Project: developing an inter-institutional network of support for instructors of the undergraduate GeT course. As conveners of the GeT: A Pencil community, the GRIP lab has been keenly aware of our responsibility to observe and support the community's progress toward this second goal. After all, without the establishment and maintenance of the GeT: A Pencil community, it will be quite dif. cult to make progress toward the goal of improving the instructional capacity for high school geometry. Because of this, many of our internal reflections, conversations, and decisions at the GRIP lab have been animated by this second goal.

Internally, we have found the *forming–storming–norming–performing* model of group development (Tuckman, 1965) useful for trying to understand both what we are observing in the community, as well as what changes in the community we should anticipate to be on the horizon. When we first began together as a community in June of 2018, Tuckman's *forming* stage described our community well—with individuals engaging in introductions with a certain amount of eagerness and excitement as well as anxiety. Similarly, our first year together was fairly well accounted for by Tuckman's *storming* phase of development. In this phase, it is typical for a group to experience some amount of conflict, competition, and drop offs in participation. This is because a *storming* group has moved past simply exchanging pleasantries into the identification of real issues facing the group. While these issues have a way of surfacing conflict, they also have a way of getting a group ready for the *norming* phase—with individuals electing to work together, in spite of their differences, to collectively seek the resolution of the issues they have identified.

In the *norming* phase of development, a group learns to resolve the kinds of personality conflicts that characterize the *storming* phase by learning to accept one another as they are in order to work collectively towards a common goal. Until recently, I felt pretty confident that the GeT: A Pencil community had been mostly operating within this phase. But in some of our recent reflections about the work people are doing within the GeT: A Pencil community, I felt it quite likely that we have turned the corner in some crucial ways—with the group engaging in activities more akin with what Tuckman described as the *performing* stage of a group.

In the *performing* stage, a group benefits from previously established norms and is able to achieve high levels of success towards meeting the goals they set out for themselves. Such a group is now able to operate in ways that are fairly autonomous, needing less of the kinds of supervision or organization they originally could not muster on their own—with the organization hierarchies that may have been crucial in the

early stages being much less visible and prominent. I'd like to take this opportunity to illustrate my point by taking a brief tour of some of the work that is happening in the community.

In the last year, the Teaching GeT Working Group has elected to set aside differences in their individual courses to work together towards a common set of student learning objectives (SLOs) for the undergraduate GeT course. Although the processes the group has used have been marked by consensus-seeking from the beginning, the group's interactions make it clear that dissent is not only an allowable part of the process but a necessary and crucial aspect of their work for the continued refinement of the SLOs for broader dissemination and consumption. Some of the artifacts this group has produced can be seen in this issue's articles entitled GeT Course Student Learning Outcome #8 and GeT Course Student Learning Outcome #10, as well as the Working Group Update provided by the group's facilitator, Dr. Nat Miller. This group also recently gathered to find avenues to disseminate their work—managing to submit two conference proposals and one handbook chapter proposal in the last six months.

Similarly, the Transformations Working Group has brought together individuals to think together what it might mean to teach geometry using a transformational approach. In this group we have seen the members grow in greater intimacy as they have voluntarily elected to open up their classrooms to one another through a series of virtual classroom observations. More than that, the group elected to do this using a process called lesson study in which the group collaboratively developed, implemented, observed, reflected on, and refined a set of lessons for teaching transformational ideas using the Adinkra patterns (see Babbitt et al., 2015). Remarkably, the choice to center a set of lessons on the Adinkra patterns emerged from an idea in the group, rather than from an area of expertise that any of the faculty members had previously developed. We see this as an important indicator of the group's willingness to be vulnerable with one another and take risks—moving past the more typical norms of competing and showcasing that can sometimes go along with lesson study in the U.S. (Rappleye & Komatsu, 2017). Like the other working group, this group has begun to disseminate about their work—producing two conference proposals and posting a note on one of the AMS Blogs (Boyce et al, May of 2021). Dr. Julia St. Goar provides an update about the current happenings of the group in this issue's Working Group Update.

We also note that for the first time since the GeT: A Pencil community formed, we, as members of the GRIP lab, have been able to make time to facilitate a series of workshops within the community. Up until now, we have had our hands full handling the administration and organization of the community—expending most of our effort making sure that things are running smoothly. But this past year, at the request of the community, we engaged in the development of items for assessing the SLOs—piloting those items with a set of GeT students this spring. This summer, we felt the other two working groups were operating autonomously enough for us to have the needed bandwidth for our team to run a series of workshops focused on engaging instructors with those items as well as the responses we gathered to them this spring. More information about those items and what we are learning through those workshops can be found in this issue's articles entitled "From Theory to Practice: Development of the SLO Items"(Ion & Herbst, 2021) and "A Deeper Dive into an SLO Item: Examining Students' Ways of Reasoning about Relationships between Euclidean and Non-Euclidean Geometries " (Herbst & Ion, 2021).

Finally, alongside the evidence we see in these bodies of work, the interactions between instructors also provide evidence that we have, indeed, turned a corner as a community. In this issue, Dr. Carolyn Hetrick, a recent graduate whose research focuses on community organization and a postdoctoral scholar at the GRIP lab, shares about some of the changes she has observed in her re-entry into the community after being away for several years.

Our community's shift into the *performing* stage of development has some important implications for our future work—including the creation of some exciting opportunities we see emerging on the horizon. One of the opportunities that we envision is a conference with a different kind of participation from GeT instructors than we envisioned for the June 2018 conference. For example, we can imagine a future conference that is organized by a committee made up of members from the GeT: A Pencil community, rather than planned solely by the GRIP. If you are interested in potentially playing a role to help shape a future conference, let us know.

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# GeT Course Student Learning Outcome #8

by Teaching GeT Working Group Members

**Be able to carry out basic Euclidean constructions and to justify their correctness.**
Geometric constructions support the curriculum in a GeT course and the development of mathematical thinking in several essential ways:

- they provide a natural place for making a mathematical argument;
- they encourage practice in using precise mathematical language when describing a construction;
- they provide the students a sense of where assumptions in building mathematical systems come from;
- they provide openings for discussion of the historical development of geometry, especially the work of Euclid;
- and they give students experience with the curriculum they will be expected to teach.

For these reasons, constructions remain an essential part of any GeT course curriculum. It should be noted that within the context of geometry, the term "construction" most often refers to traditional straightedge and compass constructions, and this is generally the assumption that we make here. However, the term can be used more generally to include other tools and manipulatives such as paper-folding (e.g. using origami or patty paper) or MIRAs™. In this context, using an external tool for the creation of an ideal geometric object whose success is argued rigorously might be reasonably considered a "construction." Indeed, the Common Core Standards for School Mathematics includes the following in its "Congruence Standard":

> G-CO.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

A GeT instructor giving even a cursory treatment of constructions should consider including the constructions listed above in CCSSM *G-CO.12* in their course. Having students discover the constructions themselves can be a powerful mathematical experience; having them present the constructions to the class offers opportunities for practice in using mathematical terminology and in making rigorous and logical arguments. The question, "How do we know that this construction is correct?" arises as a matter of course. In this way, constructions naturally reinforce SLO #1 (Proof).

Constructions (especially those using straightedge and compass) support SLO #7 (Euclid's *Elements*) as well. Not only are many of the "standard" constructions listed above given in Book I of the *Elements*, they also illustrate the discipline of mathematics as put forth by Euclid. The standards Euclid used to build geometry form the basis for the current standards of rigor used in mathematics. In building geometry, Euclid set out to establish results, many of which were known during his time, by starting with as few assumptions as possible. By viewing geometric results as constructions from two simple tools, namely a straightedge and a compass, Euclid was able to create the foundation on which all his propositions rest. Because of his approach, Euclid had to rely heavily on the power of logical reasoning. Moreover, he needed a good grasp of concepts/definitions in order to convince himself and others that his constructions were able to do what they were purported to do. Employing Euclid's approach to building geometry allows us to view a field of mathematics (geometry, algebra, analysis, etc.) as a unified whole, and it has provided us with numerous beautiful results, many of which have found practical applications in the world.

Instructors looking for ways to incorporate transformations (SLO #10) into their GeT course will find that constructions offer a useful pathway, as the fundamental transformations (reflection, rotation, translation) can be represented as straightedge and compass constructions. In addition, for those GeT courses that specifically emphasize transformations, constructions offer opportunities for mathematically rich explorations. As an example, an instructor could ask students to construct the composition of two reflections of a triangle across distinct lines (either parallel or intersecting). Identifying the single rigid motion that has the same effect requires careful construction, precision of language, and the use of cases. Nevertheless, the problem is an accessible one, and it can lead naturally to a discussion of the group of isometries.

Finally, for an instructor who intends to include the exploration of models of non-Euclidean geometry (SLO 9) in their course, work with Euclidean constructions is time well spent. Since many of our models of hyperbolic and spherical geometry reside in Euclidean geometry, constructions in those models rely on Euclidean counterparts. For example, constructing a "line" through two points in the Poincare half-plane involves finding a perpendicular bisector of a Euclidean line. With the benefit of dynamic geometry software (See SLO 6), even some of the complex constructions in non-Euclidean geometry become accessible, once one understands the fundamental Euclidean constructions.

## Suggested Citation

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# GeT Course Student Learning Outcome #10

by Teaching GeT Working Group Members

**Use transformations to explore definitions and theorems about congruence, similarity, and symmetry.**

Typically, there are three different approaches from which instructors can choose in order to embed transformation geometry learning outcomes into a GeT course. Instructors may choose to teach a GeT course using a formal transformation approach; they may choose to include a dedicated transformation unit within a course that has a predominantly traditional Euclidean focus; or they may integrate transformation approaches and concepts throughout the course offering.

There are two main types of transformations that arise in GeT courses: isometries, also known as congruence transformations (mappings that preserve both angle measure and segment length) and similarity transformations (mappings that preserve angle measures and proportionality of segment lengths). The set of isometries of the plane form an infinite non-Abelian group. Moreover, each element of the group can be classified as the identity transformation, a reflection, rotation, translation, or a glide reflection, and the composition of at most three reflections can be used to generate the other isometries. Depending on abstract algebra prerequisites, instructors may choose to highlight and formalize group properties and connections to finite symmetry groups.

It is recommended that GeT instructors build on familiar function notation when introducing transformations. This may lead to a better understanding of sequences of isometries as compositions of isometries. This notation may also make it easier for students to use rigid motions to express symmetry. GeT instructors can take advantage of looking at proofs through multiple approaches (Euclidean, analytic, transformational) to deepen students' understanding of specific theorems. For example, students can be prompted to compare other strategies after proving the base angles of an isosceles triangle are congruent by using the concept of symmetry. Some GeT instructors may begin with an informal approach to the understanding of reflections and rotations and extend the concepts of symmetry to study geometric shapes using transformations (translations, rotations, and reflections) and combinations of them. The definition of congruence is then conveyed in terms of rigid motions and the definition of similarity in terms of dilations and rigid motions. The notion of a translation typically will be introduced with directed line segments and a rotation with directed angles, though some instructors might take the opportunity to more deeply explore the concepts in terms of vectors, matrices, and coordinate geometry.

Instructors may instead choose to begin solely with sequences of reflections, which can be used to generate all other isometries of the plane. Each line of the plane is associated with a reflection that satisfies two properties: (1) every point on the line is fixed by the transformation and (2) the line is the perpendicular bisector of the segment connecting any point not on the line and the point's image under the transformation. By exploring the images of points and figures resulting from sequences of reflections about parallel and intersecting lines, GeT students can discover and establish relationships with translations, rotations, and glide reflections. Instructors may choose to also have students explore, informally, orientation preserving/reversing properties and the aspects of the group structure (associativity of composition, existence of identity and inverses, and non-commutativity).

GeT instructors have reported that students sometimes struggle with understanding fixed point properties of transformations. A case in example is when a segment is rotated a specified number of degrees about a center of rotation when the center of rotation (the fixed point) is not on the segment. Even though the center of rotation is specified, some students often choose one end-point of the segment as a center of rotation and use it as the fixed point. Using activities and technology that allow students to experience multiple examples of such properties can be included in a GeT course (see SLO 6 on use of dynamic geometry software) to clarify such misunderstandings. Douglas and Picciotto (2018) provide a guide and activities for transformational proof in high school geometry for teachers and curriculum developers. They note that The Common Core State Standards for Mathematics (2010) only require using transformations to justify the triangle congruence and similarity criteria (e.g., SAS, AA) and do not specify transformational proofs beyond this. However, Douglas and Picciotto recommend a high school geometry course using both traditional Euclidean and transformational approaches, beyond congruence and similarity criteria. It is important that prospective teachers are exposed to multiple approaches so that they not only are prepared with a solid background in geometry content but also able to make informed decisions regarding the choices they make in their own classrooms.

To prepare GeT students for transformational proofs beyond triangle congruence and similarity criteria, the design principles of St. Goar and Lai (2021) may be useful. They suggest emphasizing the bidirectionality of the definition of congruence (or similarity), highlighting that transformations act on the entire figure (or plane), and encouraging GeT students to explain how and why a prescribed transformation necessarily maps one figure to another as part of their proving activity. Instructors may also choose to compare definitions based on properties of transformations with other definitions (see SLO 5 on role of definitions). For example, one could define a kite as a convex quadrilateral for which a diagonal is a line of symmetry and prove properties about its congruent sides and congruent angles that are used as definitions in a traditional approach. Other opportunities to learn transformational geometry include explorations of tessellations of the Euclidean plane as well as explorations of transformations in non-Euclidean geometries (see SLO 9 on non-Euclidean geometry).

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# A Contribution to Stewarding the SLOs: Developing SLO Assessment Items and Examining Item Responses

by Mike Ion & Pat Herbst

Over the past year, the Teaching GeT working group proposed that one way to contribute to reducing the variability in outcomes in the preparation of secondary geometry teachers would be to formulate and steward a set of ten student learning objectives (SLOs) that could be utilized by instructors of GeT courses. We recognize that the SLOs themselves are a work in progress and that at any one time we are dealing with a version of them. Precisely because of the open-text nature of the SLOs, it is important to identify the many sources of warrants that we could rely on in order to use the SLOs to build more specific curriculum or instruction, as well as improve the SLOs themselves. Important sources for the development of the SLOs have included: the mathematical domain of geometry and its history, instructors' experiences teaching geometry courses and what they have seen their students do in those courses, policy documents for the teaching of geometry in K-12 and college, mathematics education scholarship, and instructors' knowledge of research and practice in the teaching and learning of geometry at the secondary level. Those sources have supported lively discussions about what to include and how to prioritize possible inclusions. We at the GRIP thought that gathering students' work on items that elicited knowledge of the SLOs could provide another kind of warrant to support discussions about the SLOs.

Based on the SLOs v.0 produced by the Teaching GeT group, members of the GRIP Lab at the University of Michigan developed a set of open-ended assessment items that tap into GeT students' attainment of the SLOs. The intention was to have each item elicit the knowledge named in one of the SLOs, though it was apparent that item responses might also provide evidence of knowledge of other SLOs. Following the genre of other MKT assessments (e.g., Ball et al., 2008; Herbst & Kosko, 2014; Hill et al., 2004), each item describes an event happening in a high school geometry classroom —in which the teacher needed to make a decision that required the knowledge named in that SLO. For example, the following item, designed to measure SLO 1 (Proofs), asked the participant to consider the following:
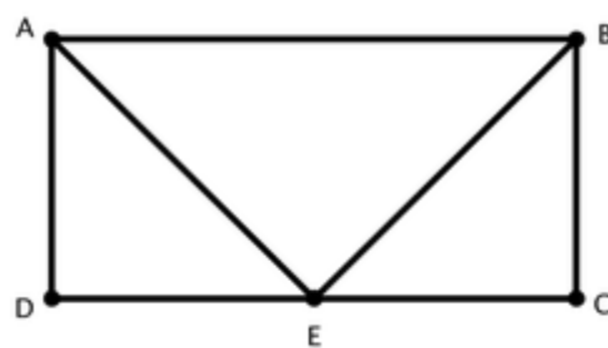


Unlike in the usual MKT-G items, the respondents did not receive a set of alternatives to choose from but were asked to compose an open-ended response and enter it in a text field.

The process through which the current set of items were created was loosely based on a set of recommended guidelines in developing measurement scales specified by DeVellis (2014, p. 105-152). In particular, as the constructs (SLOs) were already defined, the majority of the work involved scoping several items for each SLO, then choosing which of those scopes to turn into actual items, write those items, and put them through rounds of revision. The vetting of initial drafts of the items included considerations of whether the teaching scenario described in a given item (the student work, the decision the teacher had to make, etc.) seemed realistic and whether the item seemed likely to elicit a response that would be mainly driven by the participant's knowledge named in a given SLO. In the end, two items for each SLO were chosen to be administered.

These items are a first, rapid prototype of what a summative assessment might look like, created to gather data to support our collective work on the SLOs. That is, we do not yet know enough about the items to use them for consequential tasks such as appraising an individual's attainment of a specific SLO, an individual's attainment of the SLOs in their totality, or a class's average attainment of the SLOs as a proxy for the quality of the attained curriculum. The items target geometry knowledge by posing problems contextualized in tasks of teaching and make minimal assumptions about respondents' knowledge of mathematics schooling, however, they are not intended to assess knowledge of pedagogy.

While not ready to be used in any formal assessment of students or evaluation of courses, the items support the process of stewarding the SLOs by prototyping what kind of items might be needed for our whole community to document our progress in student SLO attainment. So far, we have collected student responses from seven GeT courses from the Winter 2021 term. The responses we have collected can provide an empirical basis for our community to discuss and improve the SLOs; for example, the contents students might bring up in the item responses can resonate or not with the expectations we may have had about what it would mean to attain an SLO.

In order to engage the community in that conversation, we proposed a workshop where current and prospective members of GeT: A Pencil could come and review items and students' responses to those items. Rather than work intensively over a few days like at a traditional conference workshop, and to make the workshop easier to attend, participants were asked to commit a couple of hours per week, every second week, over the summer and early fall term. For each item, they would discuss what the item seemed to assess in light of the responses and the SLOs. Participants were given access to more responses in a Canvas forum in which they continued to discuss the items. Finally, during the week of October 4th, participants had the opportunity to discuss the assessment more holistically.

In this volume and future iterations of GeT: The News!, we will provide articles that take a deeper dive into the items themselves. In these articles, we will provide an item and its intended SLO, our analysis a priori of the item, and what we heard from the instructors regarding the items, as well as how the students responded to the items in a categorized form. As we have learned from these workshops, there is much to be gained not only from the correct responses but from the incorrect or partially correct ones as well—which we will show through these writings.

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# A Deeper Dive into an SLO Item: Examining Students' Ways of Reasoning about Relationships between Euclidean and Non-Euclidean Geometries

by Pat Herbst & Mike Ion

The assessment items we developed last spring can be a resource in our collective work stewarding the Student Learning Objectives (SLOs). One way to operationalize that possibility could be to revise the items with the goal of incorporating them into a test, like the MKT-G. In such a scenario, we could envision that at some point the items could serve to provide instructors with information about how well their students attained each of the SLOs or all of them as a set. However, there is another possible use, which seems to us more compelling for the time being, and it hinges on a different interpretation of assessment: assessments that have formative, rather than summative, purposes.

Assessments like the MKT-G are often described as summative assessments; the information they provide feeds a single, synthetic score per student, that can be interpreted in terms of how much knowledge they gained over the course of the semester. As Schoenfeld (2015) says,

> *Summative assessments are examinations or performance opportunities the primary purpose of which is to assign students a score on the basis of their knowledge (p. 184).*

In that sense, gains in the MKT-G test provide information comparable to course grades. However, educators also use assessment items for formative purposes. Schoenfeld (2015) writes that,

> *Formative assessments are examinations or performance opportunities the primary purpose of which is to provide student and teachers feedback about the student's current state, while there are still opportunities for student improvement (p. 184).*

We want to examine the possibility of using the SLO Assessment items for formative purposes, namely, to provide evidence to instructors and students about students' understanding, with the goal of informing decisions the instructor might make during a class, such as what to do in the following class meeting.

Toward that end, the materials we have consulted and the discussions we have had during the summer of 2021 in the context of our assessment workshop can be quite valuable. They can help us consider what resources instructors would need in order to implement these items for formative assessment purposes. We initiate one consideration taking as an example item 15903. This item, which we transcribe below, was originally written to give students an opportunity to show evidence of attainment of SLO 9.

The student learning objective (SLO 9) about other geometries states:

> SLO 9 Compare Euclidean geometry to other geometries such as hyperbolic or spherical geometry.

We took the examples provided in SLO 9 as suggesting a distinction between Euclidean and non-Euclidean, possibly appealing to the historical efforts to prove the parallel postulate. We assumed that if a geometry course would aim for students' attainment of SLO 9, the class would likely have a discussion of the parallel postulate, its negation, and, possibly, Euclidean models of the different geometries that would ensue.

With that in mind, item 15903 reads:

**15903**

After Mr. Thompson spoke to his class about non-Euclidean geometries, some students started wondering whether everything that is true in Euclidean geometry will turn out to be false in non-Euclidean geometries. What example could Mr. Thompson offer of a theorem that is true in Euclidean geometry and in a non-Euclidean geometry?

Consistent with the genre of MKT items, the SLO items were all phrased in terms of a high school teachers' work, so as to evoke the notion that mathematical knowledge for teaching is the knowledge a teacher needs to do their job. In this case, the notion that students could ask a question that involved comparison between geometries and that the teacher would need to answer such a question was meant to simulate for the prospective teachers taking the test the work that they would need to do later on if and when they became teachers. The text of the item, particularly "some students started wondering whether everything that is true in Euclidean geometry will turn out to be false in non-Euclidean geometries," suggests that some contrasting truths might have been presented to the class. For example, the notion that, in Euclidean geometry, there is one and only one line parallel to a given line through a point not on that line might have been contrasted with how, in spherical geometry, there might not be any parallel line, or in hyperbolic geometry, there might be more than one such parallel line. Furthermore, for students to even think about the possibility that the different geometries would contradict each other everywhere, we thought it was likely that Mr. Thompson had discussed some contradictory facts across geometries. One fact that is often presented in history of mathematics textbooks is what the different geometries state for the sum of the angles in a triangle. In Euclidean geometry the triangle sum is a constant (180 degrees); in hyperbolic geometry it is not a constant, but it is always less than 180 degrees; and in elliptic geometry it is not a constant either, but is always more than 180 degrees. The context of a high school class afforded bringing in the thinking of adolescents and their penchant for playing with logical inferences by taking things to extremes. In writing the item, we thought it quite possible that after seeing two statements that said contradictory things about the same objects across different geometries, Mr. Thompson's students might consider it reasonable to pose the question they posed to him.

## Analysis a priori of item 15903

One immediate thing to notice in item 15903 is that the question the students asked Mr. Thompson implies a false generalization: even though some things which are true in Euclidean geometry are not true in non-Euclidean geometry, it is not the case that everything which is true in Euclidean geometry is false in other geometries. In order to show that generalization as false, Mr. Thompson would need a counterexample; a counterexample would be a statement which is true in Euclidean and non-Euclidean geometries. We thought item 15903 would address SLO 9 because if students had exposure to the difference between Euclidean and non-Euclidean geometries, it would show them the role the parallel postulate played in the emergence of non-Euclidean geometries. Students who had the opportunity to learn about non-Euclidean geometries might get to think of properties that rely neither on the parallel postulate nor on its alternatives to warrant their truth. The SAS criterion for triangle congruence is true in absolute geometry (geometry that satisfies Hilbert postulates except for the parallel postulate). Also, propositions that assert properties of incidence, separation, and betweenness would work across geometries.

When we shared the item with members of GeT: A Pencil, here is what we heard:

- One participant noted: "In the prompt, the student is wondering if everything that is true in Euclidean geometry will turn out to be false in non-Euclidean geometry, but the question is asking for an example of a response the teacher could give that would offer a theorem that is

true in Euclidean geometry and is also true in non-Euclidean geometry." The participant went on to note whether this will have an effect on the student responses, as the student is being asked to falsify a claim by giving an example of something that is true.

A second participant noted that students will need to understand what makes Euclidean geometry Euclidean and be familiar with examples of non-Euclidean geometries.

- Another participant noted that students would need to have a working knowledge of at least a few examples of non-Euclidean geometry, including examples of at least one theorem that is true in both Euclidean and non-Euclidean geometries.

- An additional participant noted that students should know what is meant by a "theorem." "Specifically, they would have to give some thought to the following question: "is an axiom of a theory also one of its theorems?" (From the perspective of formal logic, the answer to this question is 'Yes', but from the perspective of standard usage the answer is 'No'.)"

- During our in-person discussion, this conversation continued about whether in the GeT course there is a need to distinguish between axioms and theorems. For example, is an axiom a theorem? Some GeT instructors shared that they make this explicit for their students, while others noted that they do not. Some instructors are concerned that their students would feel like they would not be allowed to state an axiom.

- Another point of concern was whether or not the item needed to ask for a 'theorem' that is true in Euclidean and non-Euclidean geometry. One participant suggested that instead of asking "What example could Mr. Thompson offer of a **theorem** that is true in Euclidean geometry and in a non-Euclidean geometry" we could instead ask, "What example could Mr. Thompson offer of **something** that is true in Euclidean geometry and in a non-Euclidean geometry?". This participant claims that this would target the same knowledge of the SLO and whether non-Euclidean means the negation of Euclidean.

The first two bullets seem to suggest the item has face validity as part of an assessment of whether students have attained SLO 9. The third, fourth, and fifth bullets, however, suggest sources of noise in the item. The notion of a theorem, for example, brings some noise. On the one hand, the third bullet contrasts theorems with axioms, noting that logically they are similar—indeed, both axioms and theorems are declarative statements, but they are different in usage. For the question, it is important that students would think of propositions that are proved within a theory making use of axioms (i.e., theorems) and not propositions that are postulated as true to build the theory (i.e., axioms). Otherwise, not only would the question be too easy (e.g., students could bring up the axiom that two points determine a line as an example that some statements are true across Euclidean and non-Euclidean geometries), but it would also fail to tap into the interpretation of the sense of logical necessity owing from axioms to theorems called up by the question (i.e., a set of axioms defines a geometry by necessitating the truth of a set of theorems, but this does not mean that all the axioms are needed to prove all the theorems, and so if two theories have an overlapping set of axioms, as Euclidean and non-Euclidean geometries do, it is quite possible that some theorems would be true in both). To address the issue raised in the penultimate bullet, the question meant to signal, indeed, that providing an axiom would not be an answer to the question–as what students had said to Mr. Johnson, that such things would "turn out to be false," pointed to the truth value of the proposition at stake being something dependent on something else rather than postulated by choice. On the other hand, the meaning of the word theorem is not merely that of a declarative proposition that has been proved. Theorems are special propositions, deserving of recognition; for example,they conclude an investigation or present a frequently used result. Along those lines, there are status differences among declarative propositions that can be proved—theorems, lemmas, propositions, observations, and corollaries may be logically created equal but are not mathematically equal. Indeed, some of them are provable but not proved because their proof is trivial. Even Euclid distinguished the statements he proved among theorems and scholia. However, theorems do not only have special status; they also often have names or shorthands that refer to them. This is especially on point here because many theorems of Euclidean geometry do not have names or shorthands that will make them memorable; students recognize the Pythagorean theorem and maybe the base angles and exterior angle theorems, but many of the theorems that would answer the question may not have enough of those accoutrements to be remembered as theorems. Furthermore, the properties of incidence, collinearity, and separation which could answer the question were not historically theorems for Euclid but rather assumptions that later geometers made explicit. Pasch's Theorem, for example, says that if three points are not on a line and a line passes through the segment determined by two of them, the line will also pass through one of the two other segments determined by the three points. Within Hilbert's axioms for Euclidean geometry, Pasch's Theorem is a theorem which is true across geometries; yet when Pasch proposed it, he did so as a way to show the gaps in Euclid's axioms—as the Theorem cannot really be proven from Euclid's original axioms. Could students have brought up Pasch's theorem as an example that Mr. Thompson could use? Maybe, but unlikely. The example of Pasch's Theorem suggests that beyond the status differential among declarative propositions, there are historical developments used in distinguishing Euclidean and non-Euclidean geometry that could get in the way of students identifying a theorem that would be true across Euclidean and non-Euclidean geometry. Indeed, the status and historical confounds of the word *theorem* complicated the question too much; students who knew different geometries but did not have an example handy might be confused as to what would count as an example. It would be likely for students to answer that question correctly in a test if an example had been covered in their class but less likely if they had to come up with an example on their own.

The last bullet is particularly interesting in regard to the kind of assessment one is doing. The word "something" could be problematic in the context of a test or a written, summative assessment: Students might not necessarily think of declarative statements as the "somethings" to look for. The word *something* would also sound so informal and opaque that students who provided non answers (e.g., "sphere in both") might have a point to complain about in the summative assessment setting. As a result, evaluating responses might be tricky. However, in the context of a formative assessment, done in class, there might be an opportunity for the instructor to start by asking for "something" and cue students to think of declarative statements that can be proved across different geometries as the target.

## Analysis of student responses to the items

When we collected responses during Spring 2021, we found that, of the 42 student responses, 31 responses were non-trivial—that is, responses that provide some evidence of effort or knowledge of how to solve the problem. The student responses show a variety of ways in which students might relate to the question and to the distinction between Euclidean geometry and other geometries. After looking through the responses, we classify them in the following way:

Category 1: little to no evidence that the student was exposed to the knowledge of SLO 9 and some evidence that the student was swayed by an interpretation of the word "theorem," which was more speci. c than just a provable declarative statement. Five students responded to this question by naming the Pythagorean Theorem (a "common" theorem).

In these responses, most of the responses simply wrote "the pythagorean theorem" or something very similar. One response (A6) noted, "the Pythagorean Theorem and resultant distant [sic] formula hold in both Euclidean and non-Euclidean geometry." We are unsure whether the students had any exposure to the knowledge called forth by SLO 9 from these responses or if they were just naming a theorem that they knew; however, they did read the question clearly and provided a **theorem**, something that was not necessarily the case in other responses.

Category 2: some evidence that the student was exposed to the knowledge called forth by SLO 9. Three students showed knowledge of triangle angle sum properties in different geometries which we take as providing some evidence that the student was exposed to the knowledge called forth by SLO 9.

In these responses, there were references to the angle sum properties of triangles. The properties of how triangle sums differ across various non-Euclidean geometries is one of the basic facts covered in learning about those geometries. However, the knowledge the students recalled was generally incorrect. For example, responses A7 and A11 said something similar to "triangles sum to 180," which is not true in the historical examples of non-Euclidean geometries. However, students' responses were not limited to those geometries. Statement A51 ("A triangle's angles still add up to 180 degrees in taxicab geometry.") provided facts about triangles in taxicab geometries; this student attempted to give an example of a fact that is **true** in both Euclidean and a non-Euclidean geometry, but this fact is not true in the formulation of taxicab geometry where the sum of the angles is 4t-radians.

Category 3: little to some evidence that the student was exposed to the knowledge of SLO 9. Students named objects or properties of mathematical objects without any mention of explicit geometries.

Twelve responses were classified in this category. In these responses, students either named a mathematical object or properties about mathematical object(s) without explicitly naming the non-Euclidean geometry. Some examples of responses that name mathematical objects are A12 ("parallel lines"), A16 ("a straight line"), A19 ("sphere in both"), A26 ("hyperbolic shapes"), and A36 ("Parallel lines exist?..."). Some theorems can be proven about these objects, so it is possible that the students were remembering isolated bits of the knowledge associated with SLO 9, but the student did not provide a theorem nor a non-Euclidean geometry. One response, A35, states, "the fifth postulate is still true," which is incorrect, as the fifth postulate is only true in Euclidean geometry. The rest of the responses A17 ("Theorem 1.2 that states two lines have at most one point in common."), A23 ("A straight line segment can be drawn joining any two points."), A28 ("the area"), A29 ("The angle between perpendicular lines remains 90 degrees in non-euclidean geometry."), A39 ("the definition of a circle"), and A55 ("Def of line") deal with properties or definitions of mathematical objects, yet do not name a non-Euclidean geometry.

Category 4: ample evidence that the student was exposed to the knowledge associated with SLO 9. These were (mostly) correct responses.

Ten responses were classified in this category. They include students who correctly provided a theorem that holds in both Euclidean geometry and a non-Euclidean geometry. Additionally, these students were explicit about which non-Euclidean geometry the theorem holds in. These responses include references to the intersections of lines in Euclidean and hyperbolic geometries (A22, A25, A32, A38). There were two responses that looked at properties of parallel lines across Euclidean and an explicit non-Euclidean geometry (A24, A49). Statements A15 ("The SAS theorem is used in both types of geometry.") and A20 ("Proving congruency can work in both.") about triangle congruence are on the right track. However, A20 could have been more specific about a theorem and a particular non-Euclidean geometry, and A15 needed to be clear which non-Euclidean geometry SAS theorem holds in (e.g., SAS congruence is true in hyperbolic and spherical geometry but not in taxicab geometry). Statements A13 ("area of a rectangle in taxicab geometry") and A57 ("taxicab equilateral triangles are not always equiangular.") are examples of correct statements that need to be rewritten to answer the question correctly. Lastly, one response was almost correct (A56), in which a student noted, "I would explore any theorem in hyperbolic geometry that doesn't require the parallel axiom," but the student was not explicit about which theorem to choose.

## Instructor interpretations of the student responses

When members of GeT: A Pencil saw the student responses, we heard the following reactions/interpretations in the forum.

- Forum participants thought it was remarkable that a large number of items (11) named postulates or denitions.
- A24 ("In Euclidean and Hyperbolic, two lines perpendicular to the same lines are parallel.") and A51 ("A triangle's angles still add up to 180 degrees in taxicab geometry.") name common theorems in hyperbolic and taxicab geometries, respectively.
- All participants agreed that A24 was the strongest response: "in Euclidean and hyperbolic, two lines perpendicular to the same lines are parallel."

- One participant noted that A56 ("I would explore any theorem in hyperbolic geometry that doesn't require the parallel axiom.") is correct, while not providing a concrete theorem.
- One participant noted that A17 ("Theorem 1.2 that states two lines have at most one point in common."), A22 ("An example Mr. Thompson can give is that in both Euclidean Geometry and hyperbolic Geometry the intersection of two lines is at most one point, A32 ("In both Euclidean and hyperbolic geometry there is at most one point at the intersection of two lines."), and A38 ("There is at most one point for the intersection of two lines in Euclidean and hyperbolic geometry.") make the same claim, which is true in neutral geometry and is definitely a theorem (in the sense of "not an axiom"). A25 ("Lines that intersect in Euclidean geometry intersect [at] exactly one point. This can also occur in hyperbolic geometry.") makes the same claim but is marred by the phrase "can also occur;" it should be "is also true."
- Instructors noticed students naming the Pythagorean Theorem, but noted that this is not a theorem that is true in neutral geometry.

## Way Forward

As we move forward with this work, we want to hear your feedback and thoughts on what we have written here. We have heard from the participants in the workshop that these items or modifications of these items could serve as formative assessment tasks in the GeT courses. The GRIP team can serve as support, providing resources for the teaching of lessons using these tasks. Additionally, GeT instructors could work collaboratively, providing their students with the same tasks, and then come together to reflect and learn from each other on how the tasks helped elicit knowledge of the SLOs from their students.

We think that the analysis a priori as well as the categories of student responses may be helpful for instructors to use these items with formative purposes. We would love to know what you think about that and whether we could provide other things to support instructors as they use the items with their students. Additionally, if instructors who use items like 15903 collected their students' work, it could help our community continue learning about possible student responses through scans of de-identified student work uploaded to our Canvas site.

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## Suggested Citation

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# GeTing Each Other: A Reflection on Inquiry as Orientation in the GeT: A Pencil Community

by Carolyn Hetrick

In 2018, as a doctoral student researching community-based methodologies and educational advocacy, I frequently wondered about institutional, professional, and interpersonal resources and constraints that people negotiated in collaborative educational work. Particularly, I questioned how our current institutional and professional infrastructures might impact collaboration across differences: in institutions, in communities, and in disciplinary traditions and obligations. It was with this curiosity, but as an outsider to mathematics and mathematics education, that I attended the inaugural GeT A Pencil conference in June 2018. When inviting me to observe the conference, Dr. Herbst and Dr. Brown (neé Milewski) had asked me to think about my participation as ethnographic in nature. In other words, as a relative outsider to the communities, contexts, and knowledge at stake, I could immerse myself in the culture(s) of the conference. In theory, better understanding the milieu of the nascent GeT community could illuminate opportunities and challenges for supporting and sustaining its members' future collaborative work—particularly work that could meaningfully engage contextual differences and exigencies.

Preparing for my observation, mentors and colleagues in mathematics education shared some background resources with me. It was through these resources that I was introduced to the nomenclature of "the Math Wars" (Schoenfeld, 2004)—a narrative that, at the time, I understood through the lens of my previous experiences researching educational policy and community activism, in which ideologies are often explicit and oppositional. Given this perspective, I attended the 2018 conference with some notion that the potential differences between stakeholders would be more chasms than gaps.

Instead, my observations at the conference led me to think about what Lortie (1972) called "the persistence of separation" in educational institutions (p. 15). Disciplines parsed into term-long curricula, classes divided between teachers and conducted in contained spaces, and few institutional resources to support sustained collaboration are all, at least in some ways, symptomatic of the "cellular" structure (p. 13) of educational institutions and professions—including teaching being located in the cellular unit of the classroom. Ultimately, Lortie argued, the persistence of separation across the infrastructure and professional work of education has become an entrenched feedback loop, limiting opportunities for substantive, systemic change and development. While Lortie's analysis was particularly focused on teaching in the context of public K-12 schools, his work pointed to the complexities and possibilities of using an ecological perspective to engage questions about professional development and curricular change. Critically, such complexities and possibilities include dually attending to (a) the dynamic relationships and connections between people and their institutional, disciplinary, and community contexts and (b) the systemic relationships and connections between these various contexts.

Through my observations over the course of the 2018 GeT conference, I started to assemble a more nuanced understanding of and appreciation for the tensions that so many GeT instructors seemed to be navigating in their work—both within their individual contexts and across their shared contexts. Many attendees averred their multiple professional obligations as GeT instructors: to pre-service mathematics teachers, to mathematics students, to their departments and institutions, to the beauty and rigor of mathematics. Many spoke with passion and vehemence about their students' needs, their curricular scope, and the relative value of MKT items for supporting their work. In the context of group discussion of GeT course and instructor tensions, I noted some qualities of those tensions, including tensions about capacity (time, instructor load, curricular priorities); tensions about knowledge (instructor knowledge, high school teacher knowledge, knowledge hierarchies within institutional contexts, knowledge credentialing); and tensions about the disconnection involved with GeT instruction (the variability in geometry preparation within teacher education programs, the isolated nature of GeT instruction and course design, the absence of articulation between education and mathematics departments). On the final day of the conference, one attendee alluded to their concern that the gap between geometry as a mathematical discipline and geometry as an object of K-12 educational policy could present an existential dilemma to potential collaborative efforts within the GeT community in the future.

When sharing my post-conference reflections with the GRIP Lab in 2018, I recall feeling sensitive to how variable people's experiences may have been of the conference—and particularly of the conference's substantive focus on examining tensions related to institutions, students, instructors, knowledge, and experiences. My sensitivity did not presume that such variability was inauspicious for the development of the GeT community. As Bryk (2015) detailed in his model of networked improvement communities, variability of contexts and people's relationships to those contexts is unavoidable and certainly not inherently negative. Instead, Bryk explained, for educators and researchers desiring sustainable "improvement" (i.e., more accessible GeT course resources), inquiring about how "various work processes, tools, role relationships, and norms [can] *interact more productively under a variety of conditions*" (2014, p. 473, emphasis original). This perspective connects with some of my larger wonderings about the emerging culture of the GeT community. Born (2014) posited that all people relate to community in one of three main ways: turning away from others, turning against others, and turning toward others (p. 15-16). According to his framework, the quality of a community corresponds to its members' modes of engagement: when people turn away from others, shallow communities are formed; when people turn against one another, fear-based communities are formed; when people turn toward one another, deep communities are formed. In terms of this framework, only one option would undergird a story about the GeT community engaging tensions in a way that challenged the persistence of separation in education; the other two would simply reinforce and exacerbate such separation. From the ecological perspective of educational systems, a key question that I (and perhaps others) had was: *What dynamic contexts and relationships facilitate and sustain people in the GeT community in turning toward one another?*

This is one of the questions I have asked while familiarizing myself with the GeT working community—including members' current and past activities—over the past four months. I think there is good reason to believe that the work of the GeT community—members' continued participation, leadership, reporting, and visioning—is a live-action answer to that question. In conversation with physics professor Arthur Zanjoc, educational philosopher Parker Palmer (2010) named the correspondence between qualities contributing to integrity in scholarship and qualities contributing to integrity in community. Palmer wrote, "Those of us who understand inquiry, for whom it is a way of being in the world, can use our understanding to reach across gaps, ask each other questions that matter, listen with care, and find our way toward personal and communal action" (p. 133). Collaborative inquiry is a way of turning toward one another in our efforts to know and reckon with our personal and collective realities. I have witnessed evidence of this kind of inquiry in the GeT working groups, instances when people neither abandoned their individual contexts and obligations nor imposed those contexts and obligations as requisite premises for their engagement.

In the Transformations working group, I have observed GeT members engaging questions of instruction and design, including thinking together about various possibilities for engaging students in learning geometry and learning about the teaching of geometry. In one group meeting, members shared information about the people in their GeT courses and the connections they identified between their student populations and their selection and use of course materials, including textbooks. In another meeting, members of this group discussed technology tools in geometry instruction. In doing so, they telescoped between large social contexts (e.g. how the pandemic has influenced our use of technology in teaching and learning) and individual use cases related to particular mathematical tasks (e.g. designing and platforming the Adinkra lesson).

In the Teaching GeT working group, I have observed robust engagement around issues related to building professional resources for instructors of GeT courses (and particularly those for whom GeT is a new course in their teaching load). In one Teaching GeT meeting, I listened to community members share with one another about how GeT articulated with other coursework in their programs and how considering student learning objectives (SLOs) in GeT involved their thinking across their own institutional contexts and curricula. GeT members participating in the SLO item assessment workshop also often demonstrated their desire to think across their own personal context (i.e., *How would this item work with my students?*) and contexts representing more generalized tensions facing many GeT instructors (i.e., *How would this item work with pre-service teachers and mathematics majors in the same course?*).

I joined the GeT project with an understanding of the GRIP Lab team's interest in better understanding how GeT instructors share and build knowledge vis-a-vis geometry and geometry teaching. However, it has been the perseverance of the GeT community—people continuing to show up and engage with the messy realities and complexities of the ecology of GeT—that has been the most re-orienting for my own understanding of how communities of inquiry can constructively confront the persistence of separation and its attendant feedback loop between institutions and the professional work that occurs within them. I hope that, moving forward, I can learn more from all of you in the GeT community about your opinions, experiences, and insights and that we can continue to explore together the ecological implications of our collective involvement.

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# Fall 2021 Teaching GeT Working Group Update

by Nathaniel Miller

The Teaching GeT working group has been making a lot of progress! Over the summer, we wrote an article about the essential Student Learning Objectives (SLOs) for GeT classes that we have been developing. This article has been submitted to AMTE Professional Book Series, Vol. 5: *Reflection on Past, Present and Future: Paving the Way for the Future of Mathematics Teacher Education.*

In writing this paper, we revised the SLOs. Here is the latest version.

All GeT courses should prepare pre-service teachers (PSTs) to:

1. **Proofs** Derive and explain geometric arguments and proofs.
2. **Critique Reasoning**  Evaluate geometric arguments and approaches to solving problems.
3. **Secondary Geometry Understanding** Understand the ideas underlying current secondary geometry content standards and use them to inform their own teaching.
4. **Axiomatic Systems** Understand the relationship between axioms, theorems, and different geometric models in which they hold.
5. **Definitions** Understand the role of definitions in mathematical discourse.
6. **Technologies** Effectively use technologies to explore geometry and develop understanding of geometric relationships.
7. **Euclid** Demonstrate knowledge of Euclidean Geometry, including the history and basics of Euclid's Elements, and its influence on math as a discipline.
8. **Straightedge and Compass Constructions** Be able to carry out basic Euclidean constructions and justify their correctness.
9. **Non-Euclidean Geometries** Compare Euclidean geometry to other geometries such as hyperbolic or spherical geometry.
10. **Transformations** Use transformations to explore definitions and theorems about congruence, similarity, and symmetry.

- **Process Objectives**: All GeT courses should give PSTs many chances to experience and develop proficiency with the mathematical process skills of problem solving, oral and written communication of mathematical ideas, and productive collaboration. They should also have opportunities to engage with the progression of geometric exploration followed by conjecturing and construction of arguments.

Included in the submitted paper are one paragraph executive summaries of each of these SLOs. The group is still working on writing longer narratives for each of these. (Two of these are included elsewhere in this newsletter.) Eventually, our plan is for the SLOs, summaries, and narratives to all go up on a website where people can refer to them. Our thought is that these will continue to be a living document, so that as more practitioners interact with them, they can continue to evolve to better reflect the ideas and needs of the community of GeT educators.

We have been meeting every other week, and we would welcome anyone who is interested to join us.  Anyone who is interested in joining us or has any questions about this work can contact me at nathaniel.miller@unco.edu.

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# Fall 2021 Transformation Group Update

by Julia St. Goar

In the fall of 2021, the transformation group will continue its ongoing goal of collecting and creating activities and sequences of activities that contribute towards student understanding of the topic of transformations. As part of this effort, in the spring of 2021, the group created a lesson (Boyce et. al., 2021) relating to transformations and mathematical definitions. Two group members co-taught the lesson in an online class for in-service teachers, while other group members attended, and members have taught and, moving forward, will likely continue to teach the lesson in their respective courses to contribute to the discussion about and development of the lesson. This fall, the group is exploring and discussing existing transformation activities. Additionally, the group may develop and teach new activities on transformations, especially those that simultaneously contribute to student understanding of other SLOs.

The working group has an ongoing interest in axiomatic systems based on transformations that can lead to the deduction of the major congruence and similarity results. The group has explored and discussed various existing axiomatic systems, considering their affordances and possible drawbacks when teaching GeT courses and high school students. These discussions go hand-in-hand with considering and creating transformation teaching materials.

The working group meets every other week. If you are interested in joining us or have any questions about our work, please feel free to contact me at stgoarj@merrimack.edu.

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# Upcoming Events



The Teaching and Learning of Geometric Proof: Roles of the Textbook and the Teacher

Friday, November 12, 2021 at 2:00 PM ET via Zoom

Presenter: Dr. Mara Markinson

Abstract: Results from a qualitative study about proof in high school geometry will be shared. First, a Common Core geometry textbook was analyzed for its proof-related contents. Then, inservice and preservice secondary mathematics teacher participants were administered a content assessment and interviewed regarding their knowledge, beliefs, and goals for teaching proof.